CMSC201
Computer Science I for Majors

Lecture 17 – Recursion (cont)

Last Class We Covered

- Recursion
  - Recursion
    - Recursion
- Stacks
- Parts of a recursive function:
  - Base case
  - Recursive case

Any Questions from Last Time?

Today's Objectives

- To gain a more solid understanding of recursion
- To explore what goes on "behind the scenes"
- To examine individual examples of recursion
  - Fibonacci Sequence
- To better understand when it is best to use recursion, and when it is best to use iteration

Review of Recursion

What is Recursion?

- Solving a problem using recursion means the solution depends on solutions to smaller instances of the same problem

- In other words, to define a function or calculate a number by the repeated application of an algorithm

Recursive Procedures

- When creating a recursive procedure, there are a few things we want to keep in mind:
  - We need to break the problem into smaller pieces of itself
  - We need to define a "base case" to stop at
  - The smaller problems we break down into need to eventually reach the base case

"Cases" in Recursion

- A recursive function must have two things:

- At least one base case
  - When a result is returned (or the function ends)
  - "When to stop"
- At least one recursive case
  - When the function is called again with new inputs
  - "When to go (again)"

Code Tracing: Recursion

Stacks and Tracing

- Stacks will help us track what we are doing when tracing through recursive code
- Remember, stacks are **LIFO** data structures
  - Last In, First Out
- We'll be doing a recursive trace of the summation function

Summation Funcion

- The addition of a sequence of numbers
- The summation of a number is that number plus all of the numbers less than it (down to 0)
  - Summation of 5: 5 + 4 + 3 + 2 + 1 + 0
  - Summation of 6: 6 + 5 + 4 + 3 + 2 + 1 + 0
- What does a recursive implementation look like? What's the base case? Recursive case?

Summation Function

```
def summ(num):
  if num == 0:
    return 0
  else:
    return num + summ(num-1)
```

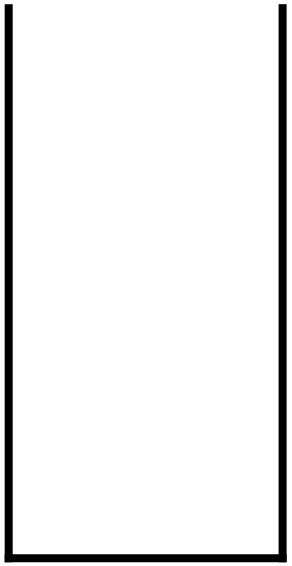
Base case:
Don't want to go below 0
Summation of 0 is 0

Recursive case and call:
Otherwise, summation is
num + summation(num-1)

```
main()

def main():
  summ(4)


def summ(num):
   if num == 0:
     return 0
   else:
     return num + summ(num-1)
```



```
main()

def main():
    summ(4)


def summ(num):
    if num == 0:
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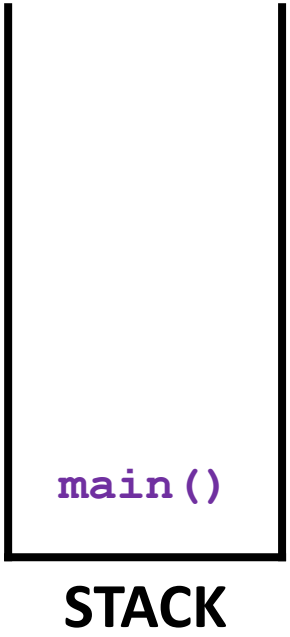


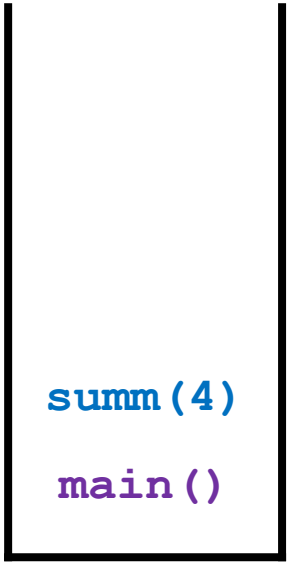
```
main()

def main():
    summ(4)

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num = 4

num: 4



STACK

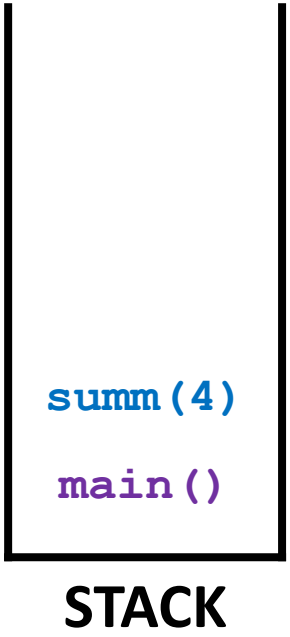
```
main()

def main():
  summ(4)
              num = 4

def summ(num):
   if num == 0:
      return 0
   else:
      return num + summ(num-1)
```

num: 4

This is a local variable. Each time the **summ()** function is called, the new instance gets its own unique local variables.



```
main()

def main():
  summ(4)
```

num = 4

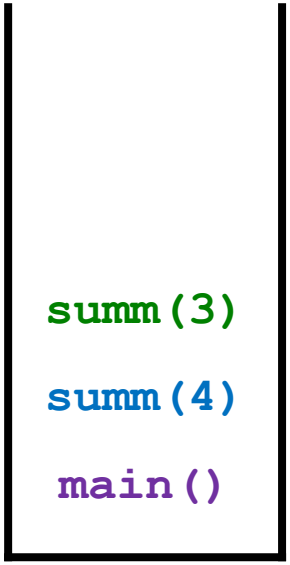
```
def summ(num):
  if num == 0:
    return 0
  else:
    return num + summ(num-1)
```

num: 4

num = 3

```
def summ(num):
  if num == 0:
    return 0
  else:
    return num + summ(num-1)
```

num: 3



STACK

```
main()

def main():
    summ(4)

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num = 4

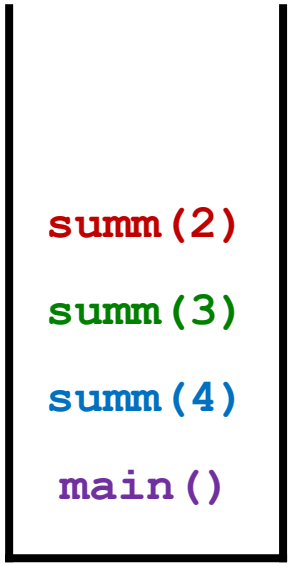
num: 4

num = 3

num: 3

num = 2

num: 2



STACK

```
main()

def main():
    summ(4)
```

num = 4

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num: 4

num = 3

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num: 3

num = 2

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num: 2

num = 1

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num +
          summ(num-1)
```

num: 1

summ(1)
summ(2)
summ(3)
summ(4)
main()

STACK

```
main()

def main():
    summ(4)

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num = 4

num: 4

num = 3

num: 3

num = 2

num = 1

num: 2

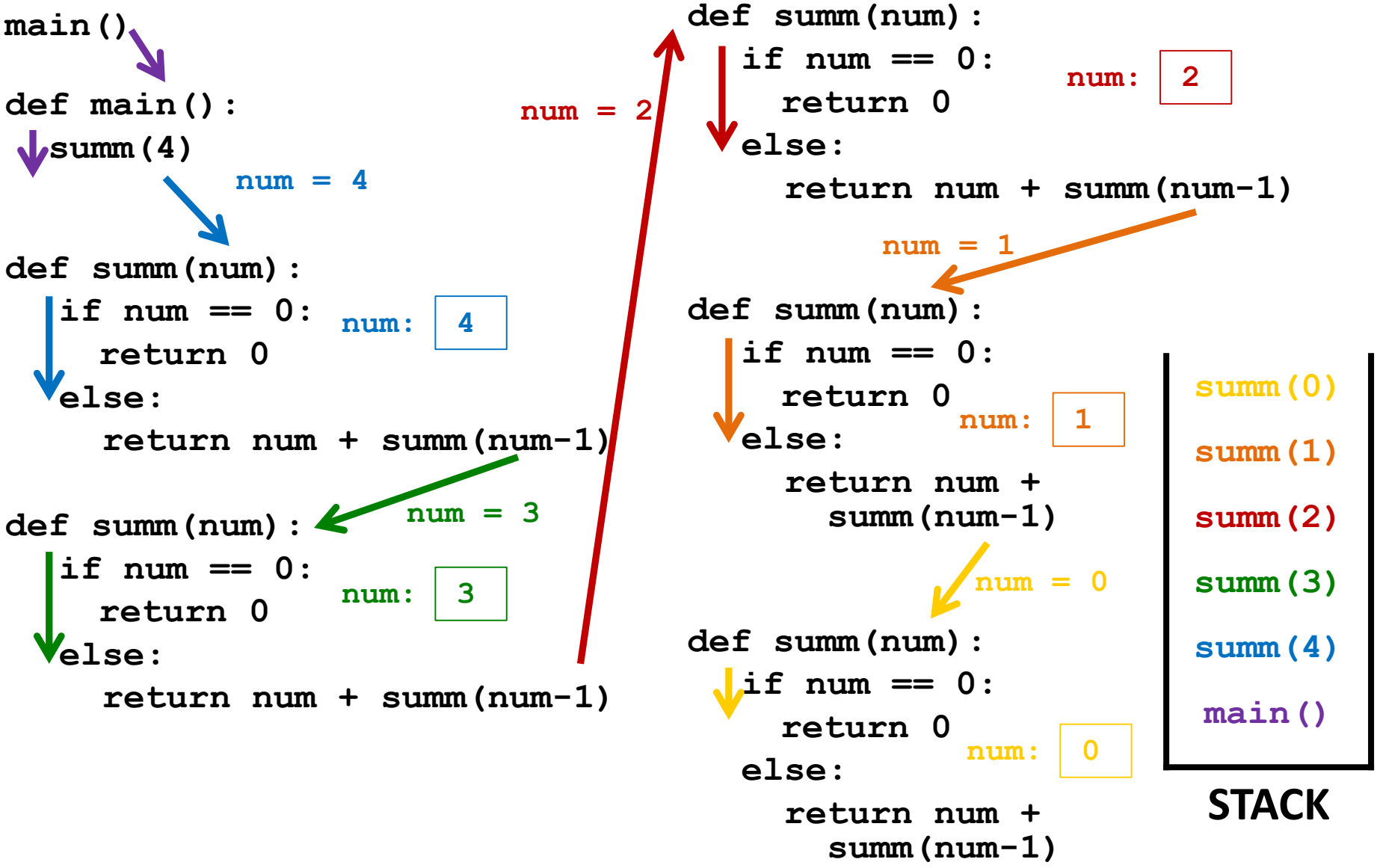
num = 0

num: 1

num: 0

summ(0)
summ(1)
summ(2)
summ(3)
summ(4)
main()

STACK



```
main()

def main():
    summ(4)

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num = 4

num: 4

num = 3

num: 3

num = 2

num: 2

num = 1

num: 1

num = 0

num: 0

return 0

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

POP!

summ(1)

summ(2)

summ(3)

summ(4)

main()

STACK

return 0

```
main()

def main():
  summ(4)

def summ(num):
  if num == 0:
    return 0
  else:
    return num + summ(num-1)

def summ(num):
  if num == 0:
    return 0
  else:
    return num + summ(num-1)

def summ(num):
  if num == 0:
    return 0
  else:
    return num + summ(num-1)
```

num = 4

num: 4

num = 3

num: 3

num = 2

num: 2

num = 1

num: 1

return 1

POP!

POP!

summ(2)

summ(3)

summ(4)

main()

STACK

return 1 + 0 (= 1)

```
main()

def main():
    summ(4)

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num = 4

num: 4

num = 3

num: 3

num = 2

num: 2

return 3

POP!

POP!

POP!

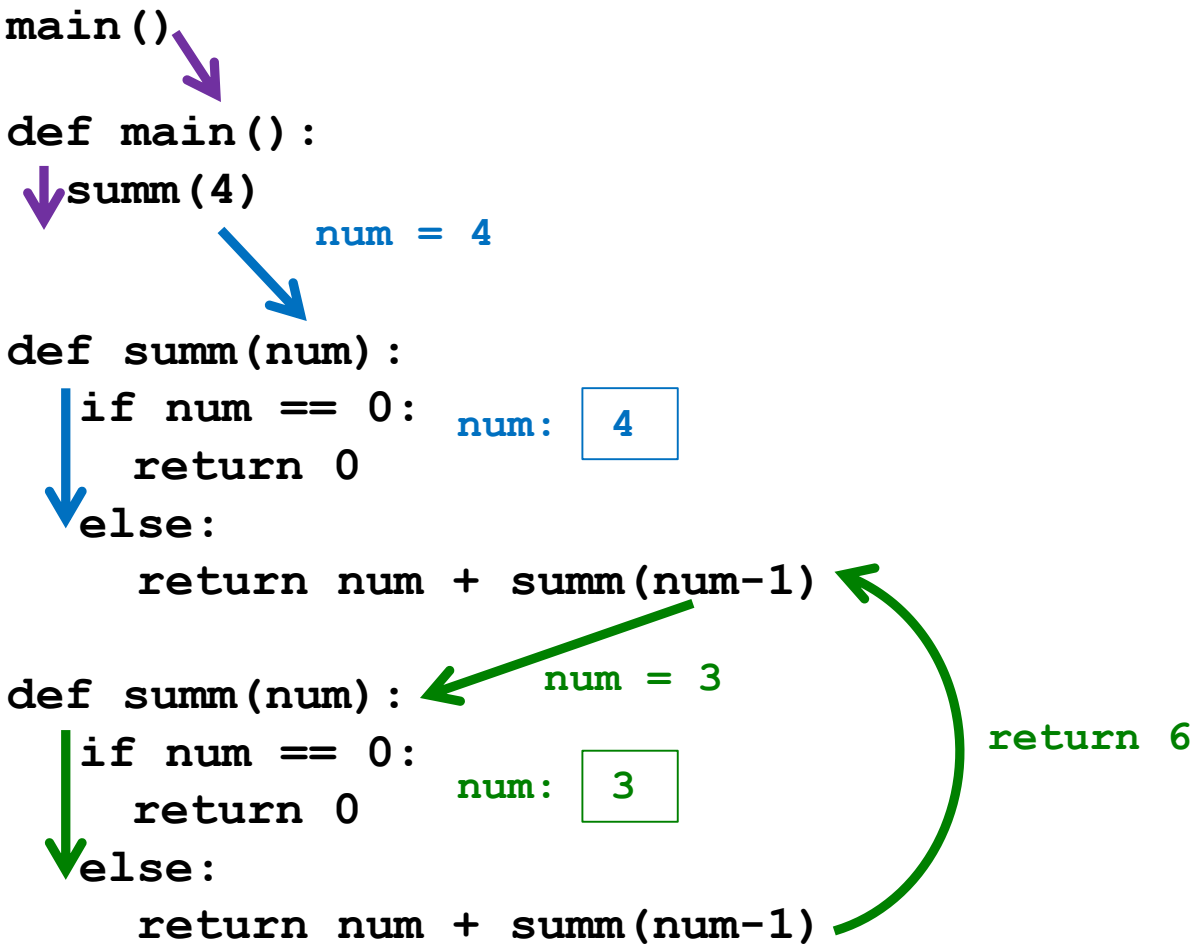
summ(3)

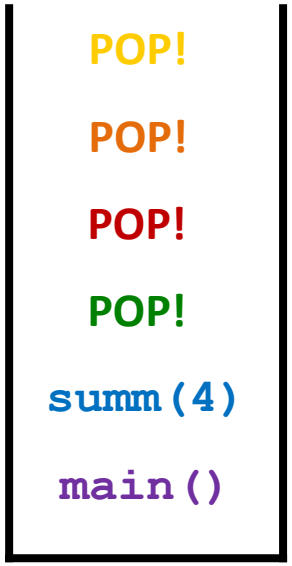
summ(4)

main()

STACK

return 2 + 1 (= 3)





STACK

return 3 + 3 (= 6)

```
main()

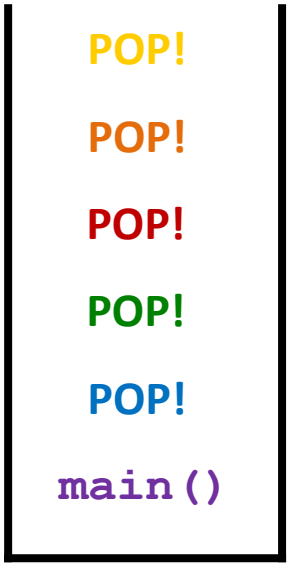
def main():
  summ(4)

def summ(num):
  if num == 0:
    return 0
  else:
    return num + summ(num-1)
```

num = 4

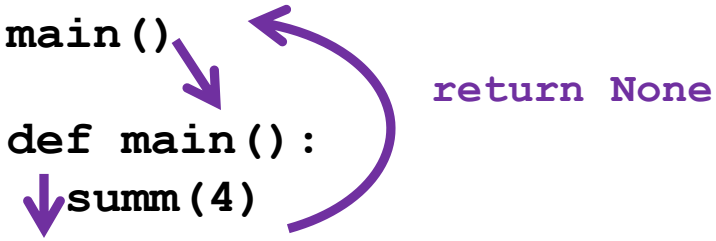
return 10

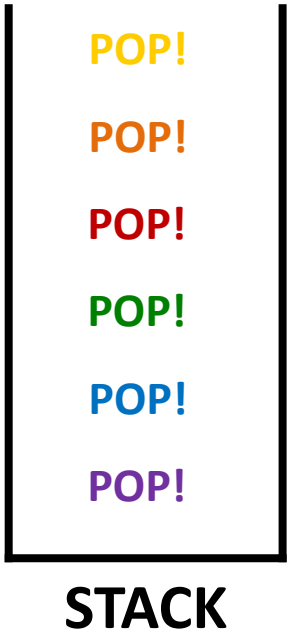
num: 4



STACK

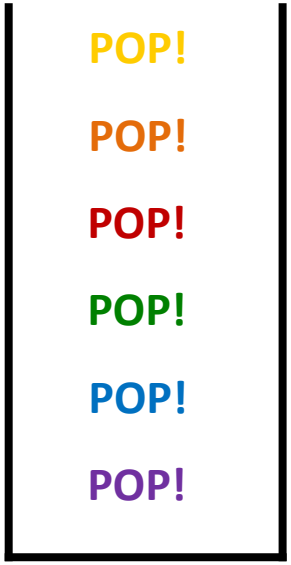
```
return 4 + 6 (=10)
```






```
return None
```

The stack is empty!




```
return control
```

Returning and Recursion

Returning Values

- If your goal is to return a final value
  - Every recursive call must return a value
  - You must be able to pass it "back up" to **`main()`**
  - In most cases, the base case should return as well
- Remember to pay attention to what happens at the "end" of a function

```
main()

def main():
    summ(4)

def summ(num):
    if num == 0:
        return 0
    else:
        num + summ(num-1)

def summ(num):
    if num == 0:
        return 0
    else:
        num + summ(num-1)

def summ(num):
    if num == 0:
        return 0
    else:
        num + summ(num-1)

def summ(num):
    if num == 0:
        return 0
    else:
        num + summ(num-1)

def summ(num):
    if num == 0:
        return 0
    else:
        num + summ(num-1)
```

num = 4

num: 4

num = 3

num: 3

num = 2

num: 2

num = 1

num: 1

num = 0

num: 0

summ(0)
summ(1)
summ(2)
summ(3)
summ(4)
main()

STACK

Does this work? What's wrong?

Recursion vs Iteration

Recursion and Iteration

- Both are important
  - All modern programming languages support them
  - Some problems are easy to solve when using one and difficult to solve if using the other

- How do you decide which to use?

Use Iteration When…

- Speed and efficiency is an issue
  - Iteration doesn't push things onto the stack
  - Can't "run out" of room like recursion does

- The problem is an obvious fit for iteration
  - Processing every element of a list (or a 2D list)

Use Recursion When…

- Speed is not an issue
- The data being processed is recursive
  - A hierarchical data structure

- A recursive algorithm is obvious
  - (Will happen with time, as you gain experience)
- Clarity and simplicity of code is important

Recursion Practice

Fibonacci Sequences

Fibonacci Sequence

- Number series
- Starts with 0 and 1

- Next number is found by adding the previous two numbers together
- Pattern is repeated over and over (and over…)

Fibonacci Sequence

- Starts with 0, 1, 1
- Next number is …?

0 1 1 2 3 5 8 13 21 34 55

89 144 233 377 610 987 …

Time for…

LIVECODING!!!

Recursively Implement Fibonacci

- The formula for a position in the sequence:

```
fib(p) = fib(p-1) + fib(p-2)
```

- What is our base case?
- What is our recursive case?

Non-Recursive `exp()`

- How do we create an exponentiation without using the exponentiation operator?
  - Given **`number`** and **`power`**, we can use code like this:

```
ans   = 1
count = 1
while count <= power:
    ans *= number
    count += 1
```

Transform this into a recursive function

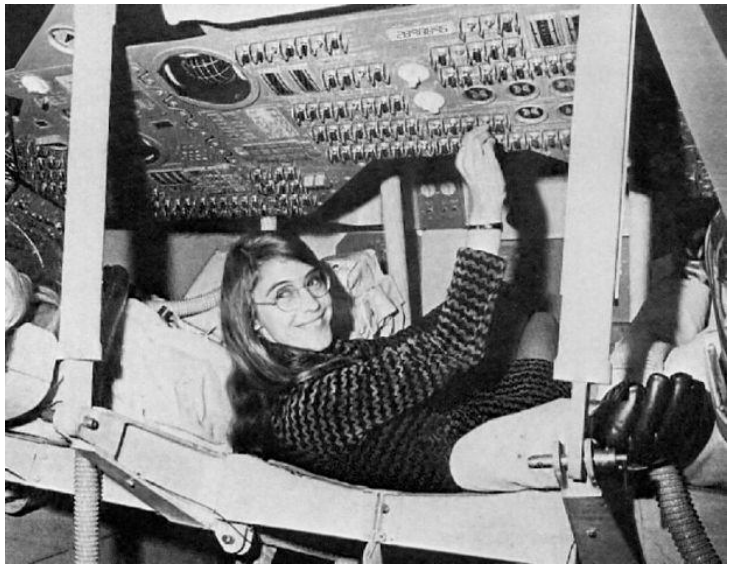
Recursive Answer

```
def recExp(number, power):
    # BASE CASE
    if power == 0:
        return 1
    # RECURSIVE CASE
    else:
        return number * \
            recExp(number, power - 1)
```

– There are other correct answers; this is just one

Daily CS History

- Margaret Hamilton
  - Created software for space flight!
    - Apollo 8
      - First to leave orbit
    - Apollo 11
      - Moon landing
  - Invented the term "software engineering"
  - Her daughter got to play with space flight simulators!



Announcements

- Project 2 is out on Blackboard now
  - Design is due by Friday (Apr 12th) at 11:59:59 PM
  - Project is due by Friday (Apr 19th) at 11:59:59 PM
- Significantly more difficult than Project 1
  - Probably at least at 10 hour project (closer to 15)
- Second midterm exam is when?
  - April 17th and 18th

Image Sources

- Margaret Hamilton
  - https://en.wikipedia.org/wiki/File:Margaret_Hamilton_in_action.jpg